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PROF. PERRY'S PRACTICAL
MATHEMATICS.

Elementary Practical Mathematics. With Numerous Exercises for the Use of Students, and especially of Mechanical and Electrical Engineering Students. By Prof. John Perry, F.R.S. Pp. xiv+335. (London: Macmillan and Co., Ltd., 1913.) Price 6s.

DURING the past ten or fifteen years a great deal of work has been done by mathematical teachers in wiping old scores off the slate and redeveloping their teaching on more sensible and rational lines. It is well known that this revolution owes its success largely to the indefatigable exertions of Prof. John Perry. We cordially agree with many of the remarks contained in the preface to the present book. It is impossible to quote the whole of the author's attacks on the old-fashioned drudgery in algebra which has disgusted many would-be mathematicians in the past, and made it quite impossible for the present reviewer ever to appreciate anything but applied mathematics. We can only quote the first few lines:—

"Academic methods of teaching mathematics succeed with about five per cent. of all students, the small minority who are fond of abstract reasoning; they fail altogether with the average student. Mathematical study may be made of great value to the average man if only it is made interesting to him."

Now one difficulty that most teachers have experienced in developing mathematics on more rational lines has certainly been the difficulty of constructing suitable exercises, examples, and examination questions. It is not that it is intrinsically more difficult to construct practical questions than it was to devise the old "pretty" question to the effect that "if tweedledum" (meaning one jumbled mass of symbols) "shew" (never show) "that tweedledee" (an equally meaningless formula). Still, to stock a book with practical intelligent questions is so difficult a task that the author's statement, "In many cases each of the questions has taken several hours, and in some cases several days, to construct," will be well understood by everyone who has worked or tried to work on similar lines.

If Prof. Perry had only published his collection of questions under the title "Exercises in Practical Mathematics," we should have given the book our unqualified praise. Unfortunately, however, when he comes to deal with *bookwork*, we

fail to find much difference between his "practical" mathematics and the old-fashioned "academic" mathematics, except that his methods are less logical, less interesting, and less convincing than those now adopted by our best teachers.

In fact, most of the bad features of our existing methods of teaching, which the author so violently attacks in his preface, will be found reproduced in his own text. The book, to some extent, resembles a recent volume which might be called "The Fool's Calculus," and which justified this title from the way the author had made an easy subject appear difficult. Let us now examine a few points in detail.

Prof. Perry is quite correct in saying that "when calculating from observed quantities it is *dishonest* to use more figures than we are sure of," although, perhaps, this mistake might be rather described as "unmathematical inaccuracy," than as dishonesty. But the only remedy he can suggest in the case of contracted multiplication is to multiply by 8651 when he wants to multiply by 1568. Very few teachers adopt this absurd and unnecessary method. A boy who has any common sense ought to learn not only to multiply numbers the right way round, but to be able to fix the position of the decimal point in any line of the products.

His definition of a logarithm is as follows:—

"If $a^n = N$ then $n = \log_a N$ and we read this as 'n is the logarithm of N to the base a.'"

Now the average schoolboy ought to learn to multiply and divide by means of a table of logarithms long before he knows what is the meaning of a^n . Besides, the definition is not a logical one unless n is a positive integer, because the very existence of quantities with fractional indices depends for its proof on the existence of a system of logarithms. By making up successive integral powers of 1'01 or 1'0001 or 1'0000001, we can prove the existence of logarithmic scales capable of performing numerical calculations to any required degree of accuracy, and these lead to the conception of the natural scale. Prof. Perry then says that "in many important calculations we need to use Napierian logarithms, whose base is 2'71828." "Why 2'71828?" asks the intelligent student. No answer is given; and this is what Prof. Perry calls "practical mathematics." We should call it cram. But the author continues to drag in this apparently useless and meaningless symbol e throughout the book, and when it occurs in such examples as the following (p. 30):—

"If $\log_{\frac{853}{493}} + 0'9 \times \frac{836}{853} = \log_{\frac{677}{493}} + \frac{961}{677}x$, find the value of x to three significant figures"

—we fail to see the superiority of Prof. Perry's questions over the worst examples of the old Cambridge school.

A very few lines of explanation, based on the definition as a limit, would make the student take an intelligent interest in e . But having started on the wrong tack, Prof. Perry, on p. 150, fails to find a satisfactory proof of the differentiation formulæ involving e without assuming the exponential series, and by the time he uses the formulæ, on p. 189, it is too late to exhibit the significance of this important limit. An intelligent boy ought to be able to understand the compound interest law and the ordinary differentiation formulæ long before he learns how to differentiate the infinite series employed in Prof. Perry's proof. Take next the formula for the belt slipping on the pulley (p. 37). The formula $N/M = e^{c\theta}$ conveys no meaning to the student of average intelligence, and it is not the method that anyone with common sense would employ in experimental work. What he would do would be to use the formula $N/M = c^n$, where c is a constant and n the number of turns, c being found by experiment.

The same mistake is made with the radian. Prof. Perry (p. 62) expects his students to be as ready to *think* in *radians* as in *degrees*, but he conspicuously fails to impress his readers sufficiently with the *utility of the radian* in connection with the relation between angular and linear velocity and differentiation formulæ.

The chapter on algebra is a good feature, if for no other reason than the fact that existing textbooks on algebra are still so unsatisfactory. The proper method of introducing algebra is in connection with the *use of formulae*, and the converse use of formulæ naturally leads to the problem of *solving an equation*. In the conventional treatment the utility of the subject is completely ignored, and the study is presented in the form of hateful drudgery. But here, again, Prof. Perry lays stress on such problems as, "Divide a number into two parts," or "A father is 3.5 times as old as his son," of which we have had too many already.

In the chapters on mensuration, squared paper, and important curves, Prof. Perry is working on what is now well-known ground; at the same time his treatment is in many respects unsatisfactory, particularly in connection with curves. Thus we all know the importance of the cycloid in geometry, mechanics, and physics. But all that Prof. Perry does is to make the student plot this curve on squared paper by means of the equations $x = a(\phi - \sin\phi)$ and $y = a(1 - \cos\phi)$. When this is done the student knows nothing whatever about what a cycloid really is. An in-

telligent boy should learn to plot curves not only from their equations, but from their geometrical definitions; and, further, he should be trained to plot envelopes as well as loci. The mere drawing of graphs may easily degenerate into unintelligent drudgery quite as objectionable as any of the old algebraical drill of our schooldays. What is the use of asking boys such questions as the following?

"Find a value of x to satisfy

$$5.3 e^{0.104x} \sin^2 0.8x + 0.78x^{1.52} \cos x - 2.126 = 0.$$

"The student must remember that $0.8x$ is in radians, and must be multiplied by 57.296 to convert it into degrees. Ans. $x = 0.74$."

In the sections on the calculus there is not very much fault to find with the practical illustrations, and, indeed, most of them are based on fairly reasonable views. But when the author comes to establishing differentiation formulæ he falls into the error of defining a differential coefficient as the limit of

$$\frac{f(x + \delta x) - f(x)}{\delta x},$$

instead of regarding it as the limit of

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1},$$

when x_2 and x_1 both approach a common limit x , which may or may not be taken to be equal to either x_2 or x_1 . Consequently he introduces higher powers of δx , which he afterwards has to neglect, and which ought never to have come in. The alternative definition here suggested leads at once to Lagrange's remainder theorem in the form,

$$f(x_2) = f(x_1) + (x_2 - x_1)f'(x),$$

where x has some value between x_1 and x_2 .

The result is that in differentiating x^n Prof. Perry assumes the binomial theorem plus certain other assumptions not stated, whereas any pupil ought to differentiate x^n long before he has heard of the binomial theorem. In speaking of limits Prof. Perry says:—

"The plain man of common sense finds no difficulty in catching the idea. Two thousand years ago neither he nor a small boy would have had a difficulty in understanding that a hare would beat a tortoise in a race; it is the mathematical philosopher who makes a difficulty about such matters, and in these days he says that this fundamental idea of the calculus can only be comprehended by a mathematician. This would not matter if these philosophers were not entrusted with the education of youth, a trust for which all their training has unfitted them. When they come to explain the essential idea of the limiting value of $\delta s/\delta t$, they talk foolishly."

Readers of "Elementary Practical Mathematics"

will have no hesitation whatever in endorsing this statement!

While a great many notions are introduced into the text in such a way as to make them appear useless, uninteresting, and unintelligible, many of the most important points in a rational system of mathematical education receive little or no attention. Take the broad, general notion of a function, so simple that it can be explained to anyone who is sufficiently unmathematical to understand common sense. It is well illustrated in the case of the senior wrangler who entered the Stock Exchange and began to apply algebraic methods to the money market. He failed because he had omitted to take account of political considerations. Here was a case where the result was a function of a number of variables, and he treated some of these variables as constant. The senior wrangler was not sufficient of a mathematician. The mere failure to enumerate all the variables in a function occurring in everyday life represents a national loss of millions per annum. Prof. Perry says:—

"I must confess, however, that the compilers of modern school algebras must make the gods laugh over the uses to which they put this plotting of functions."

They certainly will do so when they read this book.

All this is a very great pity. Most modern mathematical teachers are only too glad to get "formula" questions for their pupils dealing with beams, expansion of steam, flow of water through pipes, electrical resistances, and other practical considerations which familiarise the student in the use of algebraic formulæ and equations. We believe they *can* get the questions they want from this book, and, on the other hand, when it comes to methods of teaching, every teacher naturally prefers his own. But if the substance of the text is a fair indication of what is meant by "practical mathematics," we agree with Prof. Perry's remark (p. xiii.), which, when quoted without its context, reads to the effect that

"The subject of practical mathematics is, I am happy to say, a subject which is not likely to commend itself to such institutions, nor are such text-books likely to be of much use to real students."

Evidently neither "academic" nor "practical" mathematics supplies exactly what is wanted. They both have one fault in common, namely, that they place difficulties before the student without any rhyme or reason. Mathematics is not in itself difficult or uninteresting; a child of three can invent a theory and notation for minus quantities without any assistance, help, or encouragement whatever. What we want is a

subject that might be better described as "common-sense mathematics." Teachers are striving after this ideal, and it is very valuable and important to see clearly, as this book shows, that for the attainment of this ideal something more than "practical" mathematics is necessary.

G. H. BRYAN.

AN EGYPTIAN DESERT.

The Geography and Geology of South-eastern Egypt. By Dr. John Ball. (Survey Department of Egypt, Cairo.) Pp. 394. With Maps and many Illustrations.

THE area described in this latest monograph issued by the Geological Survey of Egypt is the southern part of the Eastern Desert—a district little known, and inhabited only by nomad Arabs of the Ababda and Bisharin tribes.

From north to south, this country is intersected by a mountainous axis which rises to heights of from 3000 to more than 5000 feet, and consists mainly of granite, diorite, gabbro, and other plutonic rocks. Eastward from this axis a series of Wadys run down, somewhat steeply, to the shores of the Red Sea on the east, these shores being almost everywhere bordered by coral reefs, which render the coast one of the foulest in the world for shipping. Westward from the mountain axis another series of Wadys lead down more gradually to the basin of the Nile. Dr. Ball, who is an accomplished surveyor as well as a geologist, has been able to add much to our knowledge of the physiography and scenery of this almost unexplored country. While devoting his chief attention to the geological features of the district—his discussion of the petrology being especially full and well illustrated—the author has been able to supply much new and interesting information concerning the antiquities, the plant and animal life, and the inhabitants—their languages, industries, and customs.

From a very early period the district has been credited with the possession of considerable mineral wealth, especially famous having been its gold-mines and emerald workings. Dr. Ball's researches, however, do not give much support to the belief that the district may in the future become a great mining centre.

It is true that very numerous small workings scattered all over the country show how wide and persistent has been the search for gold within the area. There do not appear to have been any alluvial workings, but numerous quartz veins, intersecting all the crystalline rocks of the district, sometimes containing calcite with ores of copper and iron, yield minute quantities of gold,